Student's name

Instructor's name

Course

Date

Math Assignment 2

Solution 1

The Pythagorean theorem, also called the Pythagorean equation, is highly significant in mathematics since it serves as the basis for multiple trigonometric functions and linear expressions. According to the Pythagorean theorem, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Dawson 134). The theorem states that the square of a (a^2) plus the square of b (b^2) equals the square of c (c^2).

Therefore, the Pythagorean theorem is as follows: $a^2 + b^2 = c^2$.

Taking into account that A Δ XYZ in which $\angle XYZ = 90°$, prove that $XZ^2 = XY^2 + YZ^2$.

Proof:

The proof is based on algebraic expressions. Given ΔXOY and ΔXYZ, it is possible to determine that $\angle X = \angle X$ and $\angle XOY = \angle XYZ$, each of which equals 90°.

Thus, $\Delta XOY \sim \Delta XYZ$, according to the AA similarity principle.

If $XO/XY = XY/XZ$ and $XO \times XZ = XY^2$, then, in ΔYOZ and ΔXYZ, we can prove that $\angle Z = \angle Z$ and $\angle YOZ = \angle XYZ$. In such a case, $\Delta\ YOZ \sim \Delta\ XYZ \Rightarrow OZ/YZ = YZ/XZ \Rightarrow OZ \times XZ = YZ^2$. Hence, from the expressions provided above, we will get:

$XO \times XZ + OZ \times XZ = (XY^2 + YZ^2)$

$(XO + OZ) \times XZ = (XY^2 + YZ^2)$

$XZ \times XZ = (XY^2 + YZ^2)$

$XZ^2 = (XY^2 + YZ^2)$

Solution 2

Using the sets given, find the result of theoretical operations:

A) $(B \cup C) \setminus (A \cap B)\ \Delta\ D$;

B) $(B \setminus C) \cup A\ \Delta\ D$;

C) $(B \cup C)\ \Delta D \setminus (A \cap B)$.

The following principles should be followed to perform the multiple theoretical operations in the right order:

1) the priority of operations is defined by brackets;

2) the function of brackets is performed as the completion operation if it overlaps with one of the parts of the expression;

3) the function of the brackets is followed by the intersection operation;

4) the intersection operation is followed by the union operation;

5) the differentiation and symmetrical differentiation operations have the same priority;

6) all the operations related to sets are performed from left to right.

A) Determine the result of each operation required by the instructions:

1) $B \cup C = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 8, 10, 12, 14\}$;

2) $A \cap B = \{0, 1, 3, 5, 6\}$;

3) $(B \cup C) \setminus (A \cap B) = \{-4, -3, -2, -1, 2, 4, 8, 10, 12, 14\}$;

4) $(B \cup C) \setminus (A \cap B) \Delta D = (B \cup C) \setminus (A \cap B) \Delta \varnothing = (B \cup C) \setminus (A \cap B) = \{-4, -3, -2, -1, 2, 4, 8, 10, 12, 14\}$.

B) Perform each operation in the correct order:

1) $C = U \setminus \{2, 4, 6, 8, 10, 12, 14\} = Z \setminus \{2, 4, 6, 8, 10, 12, 14\}$;

2) $B \setminus C = B \setminus \{U \setminus C\} = \{2, 4, 6\}$;

3) $(B \setminus C) \cup A = \{2, 4, 6\} \cup A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$;

4) $(B \setminus C) \cup A \Delta D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \Delta D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \Delta \varnothing = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

C) Establish the priority of operations needed and determine the final result:

1) A∩ B = {0, 1, 3, 5, 6};

2) <A ∩ B> = Z \ {0, 1, 3, 5, 6};

3) B ∪ C = {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14};

4) (B ∪ C) ΔD = (B∪ C) Δ Ø = (B∪ C) = {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 8, 10, 12, 14};

5) (B ∪ C) ΔD \ (A ∩ B) = (B ∪ C) \ (Z \ {0, 1, 3, 5, 6}) = {0, 1, 3, 5, 6}.

As a result,

A) (B ∪ C) \ (A ∩ B) Δ D = {-4, -3, -2, -1, 2, 4, 8, 10, 12, 14};

B) (B \ C) ∪ A Δ D = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9};

C) (B ∪ C) ΔD \ (A ∩ B) = {0, 1, 3, 5, 6}.

Solution 3

The fundamental theorem of Algebra suggests that a polynomial of degree $n \geq 1$ with complex coefficients has n complex roots with possible multiplicity (Fine et al. 157).

Proof of the fundamental theorem of Algebra using Louisville's Theorem:

The proof of the fundamental theorem of Algebra is based on Louisville's Theorem stating that a bounded, complete function is constant. If we take a disk of radius R and suppose that some α exists on the disk, then that $|f(\alpha)|$ is a minimum on the disk. If $f(\alpha) \neq 0$, then for any *z,* it follows that: t $|z| \geq |R|$, $|f(z)| > |f(\alpha)|$, so that $\left(\frac{1}{f(a)}\right) > \left(\frac{1}{f(z)}\right)$. Then, $\left(\frac{1}{f(a)}\right)$ is a maximum of $\left(\frac{1}{f}\right)$. If $\left(\frac{1}{f}\right)$ is holomorphic on all of C, then $\left(\frac{1}{f}\right)$ is constant, so $|f|$ is constant, according to Louisville's Theorem. Taking into account that $f(\alpha) = 0$, there are *n* complex roots in each polynomial.

Solution 4

In a finite set, different types of functions are defined by distinct formulas and ranges.

For example, if $f(x) = \sqrt{1+x^2}$, then the interval is [0; 1]. Meanwhile, if $f(x) = \sqrt{1-x^2}$, then the interval is reduced to [-1; 1].

A quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where *a*, *b*, and *c* are real numbers and constants, and a ≠ 0.

A polynomial function is defined by a formula involving the addition, subtraction, multiplication, and exponentiation of non-negative integers. In particular, if $f(x) = x\square - 3x - 1$, then $f(x) = (x-1) * (x\square + 1) + 2x^2 - 1$.

A rational function is presented as $f(x) = \frac{x-1}{x+1}$, so that $f(x) = \frac{1}{X+1} + \frac{3}{x} - \frac{2}{x-1}$.

Solution 5

Theorem 4.2.2. states that after at most [lg(n)] comparisons of the form A [*j*] < T, the length of the current sublist is 1 (Jenkyns and Stephenson 150).

Proof:

If we take that lg(n) = Q ∈ N, it does Q iterations as a whole. If Q = lg(n), then

$Q-1<lg(n)\leq Q$ so that $2^{Q-1}<n\leq 2^{Q}$.

For any k, the following expression is appropriate:

$$2^{Q-1-k}=\frac{2^{Q-1}}{2^{k}}<\frac{n}{2^{k}}\leq\frac{2^{Q}}{2^{k}}=2^{Q-k}$$.

Applying Lemma A, we can say that if x and y are real numbers and x < y, then |x| ≤ |y|, and the length of the list after k iterations is bounded:

$L(k)\leq[2/2^{k}]\leq[2^{Q}/2^{k}]=2^{Q-k}$ where Q – k ∈ P, so 2^{Q-k} ∈ P.

$L(k)\geq[2/2^{k}]\geq[2^{Q-1}/2^{k}]=2^{Q-1-k}$ where 2^{Q-1-k} ∈ P.

Then, $2=2^{1}\leq L(Q-2)\leq 2^{2}$ and $1=2^{0}\leq L(Q-1)\leq 2^{1}=2$.

Hence, after exactly Q - 1 iterations have been done, the loop must stop after Q iterations, provided L(Q -1) = 2.

Solution 6

Determine the values of the sets A and B, given that:

A) $A \setminus B = \{a, b\}$, $B \setminus A = \{c, d\}$, $A \cap B = \{x, y, z\}$;

B) $A \cup B = \{a, b, c, d, e, f\}$, $A \cap B = \{c, d\}$, $A \setminus B = \{a, e, f\}$;

C) $A \cup B = \{a, b, c, d\}$, $A \cap B = \varnothing$, $A \setminus B = \{a\}$.

A) Since $A \cap B = \{x, y, z\}$, then x, y, and z belong to both sets A and B. If $A \setminus B = \{a, b\}$ and $(A \cap B) \cup (A \setminus B) = A$, then $A = \{a, b, x, y, z\}$. Analogically, since $B \setminus A = \{c, d\}$ and $A \cap B = \{x, y, z\}$, then $B = \{c, d, x, y, z\}$.

B) Since it is known that $(A \cap B) \cup (A \setminus B) = A$, $A \cap B = \{c, d\}$, and $A \setminus B = \{a, e, f\}$, then $A = \{a, c, d, e, f\}$. Since $B = (A \cup B) \setminus (A \setminus B)$ given $A \cup B = \{a, b, c, d, e, f\}$ and $A \setminus B = \{a, e, f\}$, then $B = \{b, c, d\}$.

C) As $A \cap B = \varnothing$, $A \setminus B = A = \{a\}$. Accordingly, $B = (A \cup B) \setminus A = \{b, c, d\}$.

Works Cited

Dawson, John. *Why Prove It Again: Alternative Proofs in Mathematical Practice*. Birkhauser, 2016.

Fine, Benjamin, et al. *Introduction to Abstract Algebra: from Rings, Numbers, Groups, and Fields to Polynomials and Galois Theory*. Johns Hopkins University Press, 2015.

Jenkyns, T. A., and Ben Stephenson. *Fundamentals of Discrete Math for Computer Science: a Problem-Solving Primer*. Springer, 2018.